

Heat & Mass Transfer Formula Sheets

Definitions

Symbol	Definition	Units
q	Flow	W (energy)
q''	Flux	W/m ² (energy)
Q	Heat generation	W/m ³
μ	Viscosity	N·s/m ²
β	Coefficient of thermal expansion	K ⁻¹
c _p	Specific heat (constant pressure)	J/kg·K
c _{pa}	Apparent specific heat (constant pressure)	J/ kg·K
u, v	Velocity	m/s
u [∞]	Free stream velocity	m/s
h	Heat transfer coefficient	W/m ² ·K
k	Thermal conductivity	W/m·K
α	Thermal diffusivity	m ² /s
δ	Boundary layer	m
ρ	Density	kg/m ³
ν	Kinematic viscosity (μ/ρ)	m ² /s
p	Pressure	N/m ²
H	Enthalpy	J/kg
ΔH _f	Latent heat of fusion	J/kg
x _A	Mole fraction of solvent	none
σ	Stefan Boltzman Constant	5.67x10 ⁻⁸ W/(m ² K ⁴)
ρ _A	Mass concentration	kg of A/m ³
C _A	Molar concentration	mol of A/m ³
M _A	Molecular weight (of A)	g/mol
n _A	Moles of A	mol
P	Total pressure	atm
ρ	Density	kg/m ³
x _A	mole fraction of A in liquid	mole fraction or kg/L
p _A	Partial pressure	atm
K*	Partition/distribution coefficient	none
r _A	Reaction rate	kg/m ³ ·s
E _a	Activation energy	J/mol
τ	Shear	Pa
μ	Viscosity	Pa·s
n ^v	Volumetric flux	m/s
K	Hydraulic conductivity	m/s
H	Hydraulic potential	m
h	Matric potential	m
s	Distance	m
k	Permeability	m ²
π	Osmotic pressure	Pa
L _p	Membrane permeability	m/Pa·s
j	Mass flux	kg/m ² ·s
D _{AB}	Diffusivity (of A in B)	m ² /s
h _m	Mass transfer/convective coefficient	m/s

E_x	Dispersion coefficient	m^2/s
γ	Surface tension	N/m
N_{A1-2}	Mass flow rate	kg/s
r_A	Reaction rate (creation/depletion of A)	$kg/m^3 \cdot s$
k''	Reaction rate constant	s^{-1}
w	Moisture per unit mass of solid	kg of water/kg of dry solids
m_s	mass of solid	kg
Bi_m	Mass transfer Biot number	dimensionless
H	Henry's constant	(varies) $mol/m^3 \cdot Pa$
k	Boltzmann constant	$1.380648 \times 10^{-23} m^2 \cdot kg/s^2 \cdot K \cdot molecule$
R_g	Gas constant	$8.314 J/mol \cdot K$

Heat Transfer

Flux Equations:

- Conduction: $q'' = \frac{q}{A}$ $q'' = -k \frac{dT}{dx}$
- Convection: $q''_{1-2} = h(T_1 - T_2)$
- Radiation: $q'' = \sigma(T_1^4 - T_{surr}^4)$

Resistance: $R_{convective} = \frac{1}{hA}$ $R_{conductive} = \frac{L}{kA}$

Governing Equations:

- Slab: $\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} = \frac{k}{\rho c_p} \frac{\partial^2 T}{\partial x^2} + \frac{Q}{\rho c_p}$
- Cylinder: $\rho c_p \frac{\partial T}{\partial t} = k \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + Q$
- Sphere: $\rho c_p \frac{\partial T}{\partial t} = k \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial T}{\partial r} \right) + Q$
- Bioheat: $\rho c_p \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial x^2} + \rho_b c_b \dot{V}_b^v (T_a - T) + Q$

Lumped Parameter: $\frac{T - T_\infty}{T_i - T_\infty} = \exp\left(-\frac{t}{m c_p / h A}\right)$ $Bi = \frac{hL}{k}$

Series Solution: $\frac{T - T_s}{T_i - T_s} = \sum_{n=0}^{\infty} \frac{4(-1)^n}{(2n+1)\pi} \cos\left[\frac{(2n+1)\pi x}{L}\right] e^{-\left(\frac{(2n+1)\pi}{L}\right)^2 \frac{at}{L^2}}$ $F_o = \frac{at}{L^2}$

Semi-Infinite: $\frac{T - T_i}{T_s - T_i} = 1 - \operatorname{erf}\left[\frac{x}{2\sqrt{at}}\right]$ $L \geq 4\sqrt{at}$

Boundary Layers: $\delta_{velocity} = \frac{5x}{Re_x^{1/2}}$ $\delta_{thermal} = \frac{\delta_{velocity}}{Pr^{1/3}}$ $\delta_{velocity} = y|_{u=0.99u_\infty}$

Dimensionless numbers:

Reynolds number	$Re = \frac{u_\infty L \rho}{\mu}$	Biot number	$Bi = \frac{hL}{k_{solid}}$
Nusselt number	$Nu = \frac{hL}{k_{fluid}}$	Rayleigh number	$Ra = Gr \times Pr$
Prandtl number	$Pr = \frac{\mu c_p}{k_{fluid}} = \frac{\mu/\rho}{k_{fluid}/\rho c_p}$	Grashof number	$Gr = \frac{\beta g \rho^2 L^3 \Delta T}{\mu^2} = \frac{\beta g L^3 \Delta T}{\nu^2}$

Freezing and Thawing:

$$\frac{\Delta H_f}{R_g} \left[\frac{1}{T_{A0}} - \frac{1}{T_A} \right] = \frac{\Delta H_f}{R_g} \left[\frac{1}{T_{pure}} - \frac{1}{T_{sol'n}} \right] = \ln x_A$$

$$t_{slab} = \frac{\Delta H_f \rho}{T_m - T_s} \left(\frac{L^2}{2k} + \frac{L}{h} \right)$$

$$t_{cylinder} = \frac{\Delta H_f \rho}{T_m - T_s} \left(\frac{R^2}{4k} + \frac{R}{2h} \right)$$

$$t_{sphere} = \frac{\Delta H_f \rho}{T_m - T_s} \left(\frac{R^2}{6k} + \frac{R}{3h} \right)$$

Error function:

z	$erf(z)$	z	$erf(z)$	z	$erf(z)$
0	0	0.55	0.5633	1.3	0.9340
0.025	0.0282	0.60	0.6039	1.4	0.9523
0.05	0.0564	0.65	0.6420	1.5	0.9661
0.10	0.1125	0.70	0.6778	1.6	0.9763
0.15	0.1680	0.75	0.7112	1.7	0.9838
0.20	0.2227	0.80	0.7421	1.8	0.9891
0.25	0.2763	0.85	0.7707	1.9	0.9928
0.30	0.3286	0.90	0.7970	2.0	0.9953
0.35	0.3794	0.95	0.8209	2.2	0.9981
0.40	0.4284	1.0	0.8427	2.4	0.9993
0.45	0.4755	1.1	0.8802	2.6	0.9998
0.50	0.5205	1.2	0.9103	2.8	0.9999

Mass Transfer

Ideal Gas Law: $p_A V = n_A R_g T$

Henry's Law: $p_A = H x_A$

Darcy's Law: $n^v = -K \frac{\partial H}{\partial s} = -\frac{k}{\mu} \frac{\partial(\rho g h)}{\partial s}$

Van't Hoff Law: $\pi = c R_g T$

Surface tension: $h = 2\gamma / (\rho g r)$

Fick's Law: $j_{A,x} = -D_{AB} \frac{\partial c_A}{\partial x}$ $D = \frac{\langle x^2 \rangle}{2t}$

Dispersion: $j_{A,x} = -E_x \frac{\partial c_A}{\partial x}$

Convection-diffusion mass transfer:
 $N_{A1-2} = h_m A (c_1 - c_2)$

Chemical kinetics:

- 0th order: $r_A = -\frac{dc_A}{dt} = k''$; $c_0 - c = k''t$
- 1st order: $r_A = -\frac{dc}{dt} = k''c$; $c = c_0 e^{-k''t}$

Governing Equations:

- General: $D_{AB} \nabla^2 c_A + r_A = \frac{\partial c_A}{\partial t}$
- Cartesian: $\frac{\partial c_A}{\partial t} + u \frac{\partial c_A}{\partial x} = D_{AB} \left[\frac{\partial^2 c_A}{\partial x^2} \right] + r_A$
- Cylindrical: $D_{AB} \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial c_A}{\partial r} \right) \right] + r_A = \frac{\partial c_A}{\partial t}$
- Spherical: $D_{AB} \left[\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial c_A}{\partial r} \right) \right] + r_A = \frac{\partial c_A}{\partial t}$

Navier-Stokes: $\rho \frac{\partial u}{\partial t} = -(u \cdot \nabla) \cdot u \rho - \nabla p + \mu \nabla^2 u$

Slab: $R_{convective} = \frac{1}{h_m A}$ $R_{diffusive} = \frac{\Delta L}{DA}$

Hollow cylinder: $R_{diffusive} = \frac{\ln(r_o/r_i)}{2\pi DL}$

Hollow sphere: $R_{diffusive} = \frac{r_o - r_i}{4\pi r_o r_i D}$

Diffusion with chemical reaction: $\frac{c_A}{c_{A,0}} = \frac{-e^{-mL}}{e^{mL}-e^{-mL}} (e^{mx}-e^{-mx}) + e^{-mx} \quad m = \sqrt{\frac{k}{D}}$

Lumped Parameter: $w - w_i = -\frac{h_m A}{m_s} (c_s - c_\infty) t \quad \frac{\partial w}{\partial t} = -\frac{h_m A}{m_s} (c_s - c_\infty)$

Series Solution: $\frac{c-c_s}{c_i-c_s} = \sum_{n=0}^{\infty} \frac{4(-1)^n}{(2n+1)\pi} \cos \frac{(2n+1)\pi x}{2L} e^{-D\left(\frac{(2n+1)\pi}{2L}\right)^2 t} \quad F_0 = \frac{Dt}{L^2}$

Semi-Infinite: $\frac{c-c_i}{c_s-c_i} = 1 - \operatorname{erf} \left[\frac{x}{2\sqrt{Dt}} \right] \quad n_s = -D \left. \frac{\partial c}{\partial x} \right|_{x=0} = \sqrt{\frac{D}{\pi t}} (c_s - c_i)$
 $I = \int_0^t n_s dt = \int_0^t \sqrt{\frac{D}{\pi t}} (c_s - c_i) dt = 2\sqrt{\frac{Dt}{\pi}} (c_s - c_i) \quad L \geq 4\sqrt{Dt}$

Boundary layers: $\frac{c_{A,s}-c_{A,\delta_{conc}}}{c_{A,s}-c_{A,\infty}} = 0.99 \quad \delta_{velocity} = y|_{u=0.99u_\infty} \quad \frac{\delta_{velocity}}{\delta_{conc}} = Sc^{1/3}$

Reynolds number	$Re_x = \frac{u_\infty x \rho}{\mu}$ $Re_L = \frac{u_\infty L \rho}{\mu} = \frac{u_\infty L}{\nu}$	Biot number	$Bi_m = \frac{L/DA}{1/h_m A} = \frac{h_m K^* L}{D_{AB}}$
Sherwood number	$Sh = \frac{h_m L}{D_{AB}} = \frac{L/D_{AB}}{1/h_m}$	Rayleigh number	$Ra_m = Gr_{AB} \cdot Sc$
Schmidt number	$Sc = \frac{\nu}{D_{AB}} = \frac{\mu/\rho}{D_{AB}}$	Grashof number	$Gr_{AB} = \frac{g\rho\nabla\rho L^3}{\mu^2}$

Convection-Dispersion (infinite fluid):

$$c(z) = c_1 e^{\lambda_1 z} + c_2 e^{\lambda_2 z} \quad \Psi = \sqrt{1 + \frac{4Ek''}{u^2}} \geq 1; \lambda_1 = \frac{u}{2E} (1 + \Psi) > 0; \lambda_2 = \frac{u}{2E} (1 - \Psi) < 0$$

Convection-Dispersion (semi-infinite saturated porous solid): $\frac{c_s-c}{c_s-c_i} = \operatorname{erf} \left[\frac{\eta}{2\sqrt{Et}} \right] = \operatorname{erf} \left[\frac{z-ut}{2\sqrt{Et}} \right]$

$$c = c^* - c^{**} = c_i \left(\operatorname{erf} \left[\frac{z-ut}{2\sqrt{Et}} \right] - \operatorname{erf} \left[\frac{z-z_0-ut}{2\sqrt{Et}} \right] \right)$$

Convection-Diffusion (stagnant gas): $\frac{(1-c_A/c)}{(1-c_{A1}/c)} = \left(\frac{(1-c_{A2}/c)}{(1-c_{A1}/c)} \right)^{\frac{z-z_1}{z_2-z_1}} \quad n_A = \frac{D_{AB} c}{z_2-z_1} \ln \left(\frac{1-c_{A2}/c}{1-c_{A1}/c} \right)$

Convection-Diffusion (over a surface): $N_{A1-2} = h_m A (c_1 - c_2)$

- over a sphere, natural: $Sh_D = 2 + 0.43Ra_m^{1/4}$
- forced: $Sh_D = 2 + (0.4Re^{1/2} + 0.06Re^{2/3})Sc^{0.4}$
- over a plate: $Sh = 0.664Re^{1/2}Sc^{1/3}$ (laminar), $0.0360Re^{4/5}Sc^{1/3}$ (turbulent)