Math Help Sheet

Definitions of Derivatives:

- $\underset{\Delta x \to 0}{Lt} \frac{\Delta y}{\Delta x} = \frac{dy}{dx} \Big\} \, when \; y(x)$
- $\underset{\Delta x \to 0}{Lt} \frac{y_{x+\Delta x} - y_x}{\Delta x} = \frac{dy}{dx} \Big\} \, when \; y(x,t)$
- $\underset{\Delta x \to 0}{Lt} \frac{(x+\Delta x) y_{x+\Delta x} - x y_x}{\Delta x} = \frac{d(xy)}{dx} \Big\} \, when \; y(x,t)$
- $\ln(1+x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \cdots \approx x$ (for small x)
- $\mathrm{erf}(n) = \frac{2}{\sqrt{\pi}} \int_0^n e^{-n^2} dn$
- $\frac{d}{dx}(\ln x) = \frac{1}{x}$
- $\int \frac{dT}{T^n} = \frac{T^{-n+1}}{-n+1}$
- $\int \frac{dT}{T} = \ln T$
- $\nabla^2 = \frac{d^2}{dx^2} + \frac{d^2}{dy^2} + \frac{d^2}{dz^2} \Big\} \; Cartesian$
- $= \frac{1}{r}\frac{d}{dr}\left(r \frac{d}{dr}\right) + \frac{1}{r^2}\frac{d^2}{d\emptyset^2} + \frac{d^2}{dz^2} \Big\} \, Cylindrical$

Solutions to Common Derivatives:

- $\frac{dT}{dx} = cT \to \boldsymbol{T = c_1 e^{cx}}$
- $\frac{d^2T}{dx^2} = 0 \to \frac{dT}{dx} = c_1 \to \boldsymbol{T = c_1 x + c_2}$
- $\frac{d^2T}{dx^2} = c_0 \to \frac{dT}{dx} = c_0 x + c_1 \to \boldsymbol{T = \frac{c_0 x^2}{2} + c_1 x + c_2}$
- $\frac{d}{dr}\left(r \frac{dT}{dr}\right) = c_0 \to r\frac{dT}{dr} = c_0 r + c_1 \to \frac{dT}{dr} = c_0 + \frac{c_1}{r} \;\; \to \boldsymbol{T = c_0 + c_1 \ln(r) + c_2}$

- Solution to $\frac{d^2T}{dx^2} - cT = 0$ is $T = Ae^{\sqrt{c}x} \;\; or \;\; T = Ae^{-\sqrt{c}x}$
    - General solution is $T = Ae^{\sqrt{c}x} + Be^{-\sqrt{c}x}$ where "A" and "B" are constants.
    - Average of a function **T(x)** over **0 ≤ x ≤ L** is given by $T_{ave} = \frac{\int_0^L T dx}{L}$
    - Solution to $\frac{d^2T}{dx^2} + cT = 0$ is $T = A\sin\sqrt{c}x$ or $T = A\cos\sqrt{c}x$
    - General solution is $T = A\sin\sqrt{c}x + B\cos\sqrt{c}x$ where "A" and "B" are constants.
- $\frac{dT}{dt} = c\frac{d^2T}{dx^2}$ can be solved by using separation of variables.

Basic Properties/Rules:

- $\int cf(x)dx = c\int f(x)dx$ ("c" constant)
- $\int k\, dx = kx + c$
- $\int \frac{1}{x} dx = \ln|x| + c$
- $\int x^2\, dx = \frac{x^3}{3} + c$
- $\int x\, dx = \frac{x^2}{2} + c$
- $\int e^x\, dx = e^x + c$
- $\frac{d}{dx}(c) = 0$ where "c" is a constant
- $\frac{d}{dx}(x^n) = nx^{n-1}$
- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\cos x) = -\sin x$
- $\int_a^b f(x)\, dx = -\int_b^a f(x)\, dx$

Integration by Parts:

- $\int u\, dv = uv - \int v\, du$
    - Choose "u" and "dv." Compute "du" by differentiation u and "v" by understanding $v = \int dv$